Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/ Advanced Subsidiary

Wednesday 22 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P38168A

1. $$f(x) = 3^x + 3x - 7$$

(*a*) Show that the equation f $(x) = 0$ has a root α between $x = 1$ and $x = 2$.

(2)

(*b*) Starting with the interval [1, 2], use interval bisection twice to find an interval of width 0.25 which contains α.

(3)

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2. $$z_1 = -2 + \mathrm{i}$$

(*a*) Find the modulus of z_1.

(1)

(*b*) Find, in radians, the argument of z_1, giving your answer to 2 decimal places.

(2)

The solutions to the quadratic equation

$$z^2 - 10z + 28 = 0$$

are z_2 and z_3.

(*c*) Find z_2 and z_3, giving your answers in the form $p \pm \mathrm{i}\sqrt{q}$, where p and q are integers.

(3)

(*d*) Show, on an Argand diagram, the points representing your complex numbers z_1, z_2 and z_3.

(2))

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3. (*a*) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix},$$

(i) find $\mathbf{A}^2$,

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$.

(4)

(*b*) Given that

$$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

describe fully the geometrical transformation represented by **B**.

(2)

(*c*) Given that

$$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix},$$

where k is a constant, find the value of k for which the matrix **C** is singular.

(3)

4.
$$\mathrm{f}(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0.$$

(*a*) Use differentiation to find $\mathrm{f}'(x)$.

(2)

The root α of the equation $\mathrm{f}(x) = 0$ lies in the interval [0.7, 0.9].

(*b*) Taking 0.8 as a first approximation to α, apply the Newton-Raphson process once to $\mathrm{f}(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

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5. $$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the matrix **A** maps the point with coordinates (4, 6) onto the point with coordinates (2, −8),

(*a*) find the value of *a* and the value of *b*.

(4)

A quadrilateral *R* has area 30 square units.
It is transformed into another quadrilateral *S* by the matrix **A**.
Using your values of *a* and *b*,

(*b*) find the area of quadrilateral *S*.

(4)

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6. Given that $z = x + \mathrm{i}y$, find the value of *x* and the value of *y* such that

$$z + 3\mathrm{i}z^* = -1 + 13\mathrm{i}$$

where z^* is the complex conjugate of *z*.

(7)

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7. (*a*) Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(2n+1)(2n-1)$$

for all positive integers *n*.

(6)

(*b*) Hence show that

$$\sum_{r=n+1}^{3n} (2r-1)^2 = \frac{2}{3}n(an^2 + b)$$

where *a* and *b* are integers to be found.

(4)

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8. The parabola C has equation $y^2 = 48x$.

The point $P(12t^2, 24t)$ is a general point on C.

(*a*) Find the equation of the directrix of C.

(2)

(*b*) Show that the equation of the tangent to C at $P(12t^2, 24t)$ is

$$x - ty + 12t^2 = 0.$$

(4)

The tangent to C at the point (3, 12) meets the directrix of C at the point X.

(*c*) Find the coordinates of X.

(4)

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9. Prove by induction, that for $n \in \mathbb{Z}^+$,

(*a*) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$

(6)

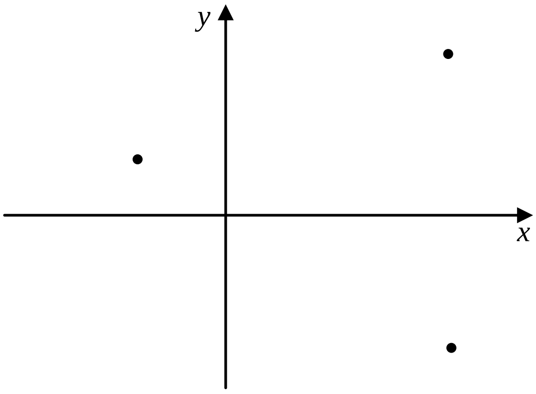
(*b*) $\text{f}(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Notes	Marks
1.	$f(x) = 3^x + 3x - 7$		
(a)	$f(1) = -1$ $f(2) = 8$	Either any one of f(1) = -1 or f(2) = 8.	M1
	Sign change (positive, negative) (and $f(x)$ is continuous) therefore (a **root**) α is between $x = 1$ and $x = 2$.	Both values correct, sign change and conclusion	A1
			(2)
(b)	$f(1.5) = 2.696152423...$ $\{\Rightarrow 1 \leqslant \alpha \leqslant 1.5\}$	$f(1.5) =$ awrt 2.7 (or truncated to 2.6)	B1
	$f(1.25) = 0.698222038...$	Attempt to find $f(1.25)$.	M1
	$\Rightarrow 1 \leqslant \alpha \leqslant 1.25$	$f(1.25) =$ awrt 0.7 with $1 \leqslant \alpha \leqslant 1.25$ or $1 < \alpha < 1.25$ or $[1, 1.25]$ or $(1, 1.25)$. or equivalent in words.	A1 (3)
			5

Question Number	Scheme	Notes	Marks
2. (a)	$\lvert z_1 \rvert = \sqrt{(-2)^2 + 1^2} = \sqrt{5} = 2.236...$	$\sqrt{5}$ or awrt 2.24	B1 (1)
(b)	$\arg z = \pi - \tan^{-1}\left(\frac{1}{2}\right)$	$\tan^{-1}\left(\frac{1}{2}\right)$ or $\tan^{-1}\left(\frac{2}{1}\right)$ or $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ or $\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ or $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ or $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$	M1
	$= 2.677945045... = 2.68$ (2 dp)	awrt 2.68	A1 oe
	$\arg z = \tan^{-1}\left(\frac{1}{-2}\right) = -0.46$ on its own is M0 but $\pi + \tan^{-1}\left(\frac{1}{-2}\right) = 2.68$ scores M1A1 $\pi - \tan^{-1}\left(\frac{1}{-2}\right) =$ is M0 as is $\pi - \tan\left(\frac{1}{2}\right)$ (2.60)		(2)
(c)	$z^2 - 10z + 28 = 0$		
	$z = \dfrac{10 \pm \sqrt{100 - 4(1)(28)}}{2(1)}$	An attempt to use the quadratic formula (usual rules)	M1
	$= \dfrac{10 \pm \sqrt{100 - 112}}{2}$		
	$= \dfrac{10 \pm \sqrt{-12}}{2}$		
	$= \dfrac{10 \pm 2\sqrt{3}\mathrm{i}}{2}$	Attempt to simplify their $\sqrt{-12}$ in terms of i,. e.g. i $\sqrt{12}$ or i $\sqrt{3 \times 4}$	M1
	So, $z = 5 \pm \sqrt{3}\mathrm{i}$. $\{p = 5,\ q = 3\}$	$5 \pm \sqrt{3}\mathrm{i}$	A1 oe (3)
(d)		Note that the points are $(-2, 1)$, $(5, \sqrt{3})$ and $(5, -\sqrt{3})$.	
		The point $(-2, 1)$ plotted correctly on the Argand diagram with/without label.	B1
		The distinct points z_2 and z_3 plotted correctly and symmetrically about the x-axis on the Argand diagram with/without label.	B1√ (2)
			8

Question Number	Scheme	Notes	Marks
3. (a)	$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$		
(i)	$\mathbf{A}^2 = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$		
	$= \begin{pmatrix} 1+2 & \sqrt{2}-\sqrt{2} \\ \sqrt{2}-\sqrt{2} & 2+1 \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct elements.	M1
	$= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	Correct answer	A1
			(2)
(ii)	**Enlargement**; scale factor 3, centre $(0, 0)$.	**Enlargement**; scale factor **3**, centre **(0, 0)**	B1; B1
			(2)
(b)	$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		
	Reflection; in the line $y = -x$.	**Reflection**; $y = -x$	B1; B1 (2)
(c)	$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$, k is a constant.		
	C is singular $\Rightarrow \det \mathbf{C} = 0$. (Can be implied)	$\det \mathbf{C} = 0$	B1
	$9(k+1) - 12k \; (= 0)$	Applies $9(k+1) - 12k$	M1
	$9k + 9 = 12k$		
	$9 = 3k$		
	$k = 3$	$k = 3$	A1 (3)
			9

Question Number	Scheme	Notes	Marks
4.	$f(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0$		
(a)	$f(x) = x^2 + \frac{5}{2}x^{-1} - 3x - 1$		
	$f'(x) = 2x - \frac{5}{2}x^{-2} - 3 \{+ 0\}$	At least two of the four terms differentiated correctly.	M1
		Correct differentiation. (Allow any correct unsimplified form)	A1
	$\left\{f'(x) = 2x - \frac{5}{2x^2} - 3\right\}$		(2)
(b)	$f(0.8) = 0.8^2 + \frac{5}{2(0.8)} - 3(0.8) - 1 (= 0.365)\left(= \frac{73}{200}\right)$	A correct numerical expression for f(0.8)	B1
	$f'(0.8) = -5.30625\left(= \frac{-849}{160}\right)$	Attempt to insert x = 0.8 into their f'(x). Does not require an evaluation. (If $f'(0.8)$ is incorrect for their derivative and there is no working score M0)	M1
	$\alpha_2 = 0.8 - \left(\frac{\text{"0.365"}}{\text{"}-5.30625\text{"}}\right)$	Correct application of Newton-Raphson using their values. Does not require an evaluation.	M1
	$= 0868786808...$		
	$= 0.869$ (3dp)	0.869	A1 cao (4)
			6

Question Number	Scheme	Notes	Marks
5.	$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}$, where a and b are constants.		
(a)	$\mathbf{A}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$		
	Therefore, $\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by both correct equations below.	M1
	So, $-16 + 6a = 2$ and $4b - 12 = -8$ Allow $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	Any one correct equation. Any correct horizontal line	M1
	giving $a = 3$ and $b = 1$.	Any one of $a = 3$ or $b = 1$.	A1
		Both $a = 3$ and $b = 1$.	A1
			(4)
(b)	$\det \mathbf{A} = 8 - (3)(1) = 5$	Finds determinant by applying $8 -$ their ab.	M1
		$\det \mathbf{A} = 5$	A1
	Area $S = (\det \mathbf{A})(\text{Area } R)$		
	Area $S = 5 \times 30 = 150$ (units)^2	$\dfrac{30}{\text{their } \det \mathbf{A}}$ or $30 \times (\text{their } \det \mathbf{A})$	M1
		150 or ft answer	A1 $\sqrt{}$ (4)
			8

Question Number	Scheme	Notes	Marks
6.	$z + 3\mathrm{i}z^* = -1 + 13\mathrm{i}$		
	$(x + \mathrm{i}y) + 3\mathrm{i}(x - \mathrm{i}y)$	$z^* = x - \mathrm{i}y$	B1
		Substituting $z = x + \mathrm{i}y$ and their z^* into $z + 3\mathrm{i}z^*$	M1
	$x + \mathrm{i}y + 3\mathrm{i}x + 3y = -1 + 13\mathrm{i}$	Correct equation in x and y with $\mathrm{i}^2 = -1$. Can be implied.	A1
	$(x + 3y) + \mathrm{i}(y + 3x) = -1 + 13\mathrm{i}$		
	Re part: $x + 3y = -1$ Im part: $y + 3x = 13$	An attempt to equate real **and** imaginary parts.	M1
		Correct equations.	A1
	$3x + 9y = -3$ $3x + y = 13$		
	$8y = -16 \Rightarrow y = -2$	Attempt to solve simultaneous equations to find one of x or y. **At least one of the equations must contain both x and y terms.**	M1
	$x + 3y = -1 \Rightarrow x - 6 = -1 \Rightarrow x = 5$	Both $x = 5$ and $y = -2$.	A1
	$\{z = 5 - 2\mathrm{i}\}$		(7)
			7

Question Number	Scheme	Notes	Marks
7.	$\{S_n =\} \sum_{r=1}^{n} (2r-1)^2$		
(a)	$= \sum_{r=1}^{n} 4r^2 - 4r + 1$	Multiplying out brackets and an attempt to use at least one of the two standard formulae correctly.	M1
	$= \underline{4.\frac{1}{6}n(n+1)(2n+1) - 4.\frac{1}{2}n(n+1)} + n$	First two terms correct.	A1
		$+n$	B1
	$= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$		
	$= \frac{1}{3}n\{2(n+1)(2n+1) - 6(n+1) + 3\}$	Attempt to factorise out $\frac{1}{3}n$	M1
		Correct expression with $\frac{1}{3}n$ factorised out with no errors seen.	A1
	$= \frac{1}{3}n\{2(2n^2+3n+1) - 6(n+1) + 3\}$ $= \frac{1}{3}n\{4n^2+6n+2 - 6n - 6 + 3\}$ $= \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(2n+1)(2n-1)$	Correct proof. No errors seen.	A1 *
			(6)
(b)	$\sum_{r=n+1}^{3n} (2r-1)^2 = S_{3n} - S_n$		
	$= \frac{1}{3}.3n(6n+1)(6n-1) - \frac{1}{3}n(2n+1)(2n-1)$	Use of $S_{3n} - S_n$ or $S_{3n} - S_{n+1}$ with the result from (a) used at least once.	M1
		Correct unsimplified expression. E.g. Allow $2(3n)$ for $6n$.	A1
	$= n(36n^2-1) - \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(108n^2 - 3 - 4n^2 + 1)$	Factorising out $\frac{1}{3}n$ (or $\frac{2}{3}n$)	dM1
	$= \frac{1}{3}n(104n^2 - 2)$		
	$= \frac{2}{3}n(52n^2-1)$	$\frac{2}{3}n(52n^2-1)$	A1
	$\{a = 52,\ b = -1\}$		(4)
			10

Question Number	Scheme	Notes	Marks
8.	$C: y^2 = 48x$ with general point $P(12t^2, 24t)$.		
(a)	$y^2 = 4ax \Rightarrow a = \frac{48}{4} = 12$	Using $y^2 = 4ax$ to find a.	M1
	So, directrix has the equation $x + 12 = 0$	$x + 12 = 0$	A1 oe (2)
(b)	$y = \sqrt{48}\,x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\sqrt{48}\,x^{-\frac{1}{2}}\left(=2\sqrt{3}\,x^{-\frac{1}{2}}\right)$ or (implicitly) $y^2 = 48x \Rightarrow 2y\frac{dy}{dx} = 48$ or (chain rule) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 24 \times \frac{1}{24t}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm k\,x^{-\frac{1}{2}}$ $ky\frac{\mathrm{d}y}{\mathrm{d}x} = c$ their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}}\right)$	M1
	When $x = 12t^2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{48}}{2\sqrt{12t^2}} = \frac{\sqrt{4}}{2t} = \frac{1}{t}$ or $\frac{dy}{dx} = \frac{48}{2y} = \frac{48}{48t} = \frac{1}{t}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{t}$	A1
	T: $y - 24t = \frac{1}{t}(x - 12t^2)$	Applies $y - 24t =$ their $m_T(x - 12t^2)$ or $y = (\text{their } m_T)x + c$ using $x = 12t^2$ *and* $y = 24t$ in an attempt to find c. **Their m_T must be a function of t.**	M1
	T: $ty - 24t^2 = x - 12t^2$		
	T: $x - ty + 12t^2 = 0$	Correct solution.	A1 cso* (4)
(c)	Compare $P(12t^2, 24t)$ with $(3, 12)$ gives $t = \frac{1}{2}$.	$t = \frac{1}{2}$	B1
	NB $x - ty + 12t^2 = 0$ with x = 3 and y = 12 gives $4t^2 - 4t + 1 = 0 = (2t-1)^2 \Rightarrow t = \frac{1}{2}$		
	$t = \frac{1}{2}$ into **T** gives $x - \frac{1}{2}y + 3 = 0$	Substitutes their t into **T**.	M1
	At X, $x = -12 \Rightarrow -12 - \frac{1}{2}y + 3 = 0$	Substitutes their x from (a) into **T**.	M1
	So, $-9 = \frac{1}{2}y \Rightarrow y = -18$		
	So the coordinates of X are $(-12, -18)$.	$(-12, -18)$	A1 (4)
			10

Question Number	Scheme	Notes	Marks
9. (a)	$n=1$; LHS $= \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^1 = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ RHS $= \begin{pmatrix} 3^1 & 0 \\ 3(3^1-1) & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ As LHS = RHS, the matrix result is true for $n=1$.	Check to see that the result is true for $n=1$.	B1
	Assume that the matrix equation is true for $n=k$, ie. $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$		
	With $n=k+1$ the matrix equation becomes $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}\begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$	$\begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$ by $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 3^{k+1}+0 & 0+0 \\ 9(3^k-1)+6 & 0+1 \end{pmatrix}$ or $\begin{pmatrix} 3^{k+1}+0 & 0+0 \\ 6.3^k+3(3^k-1) & 0+1 \end{pmatrix}$	Correct unsimplified matrix with no errors seen.	A1
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 9(3^k)-3 & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3\left(3(3^k)-1\right) & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3(3^{k+1}-1) & 1 \end{pmatrix}$	Manipulates so that $k \to k+1$ on at least one term.	dM1
		Correct result with no errors seen with some working between this and the previous A1	A1
	If the result is true for $n=k$ (1) then it is now true for $n=k+1$. (2) As the result has shown to be true for $n=1$, (3) then the result is true for all n. (4)	Correct conclusion with all previous marks earned	A1 cso
			(6)

Question Number	Scheme	Notes	Marks
(b)	$f(1) = 7^{2-1} + 5 = 7 + 5 = 12$,	Shows that $f(1) = 12$.	B1
	{which is divisible by 12}. {$\therefore f(n)$ is divisible by 12 when $n = 1$.}		
	Assume that for $n = k$, $f(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in \mathbb{Z}^+$.		
	So, $f(k+1) = 7^{2(k+1)-1} + 5$	Correct unsimplified expression for $f(k+1)$.	B1
	giving, $f(k+1) = 7^{2k+1} + 5$		
	$\therefore f(k+1) - f(k) = \left(7^{2k+1} + 5\right) - \left(7^{2k-1} + 5\right)$	Applies $f(k+1) - f(k)$. No simplification is necessary and condone missing brackets.	M1
	$= 7^{2k+1} - 7^{2k-1}$		
	$= 7^{2k-1}\left(7^2 - 1\right)$	Attempting to isolate 7^{2k-1}	M1
	$= 48\left(7^{2k-1}\right)$	$48\left(7^{2k-1}\right)$	A1cso
	$\therefore f(k+1) = f(k) + 48\left(7^{2k-1}\right)$, which is divisible by 12 as both $f(k)$ and $48\left(7^{2k-1}\right)$ are both divisible by 12.(1) If the result is true for $n = k$, (2) then it is now true for $n = k+1$. (3) As the result has shown to be true for $n = 1$, (4) then the result is true for all n. (5).	Correct conclusion with no incorrect work. Don't condone missing brackets.	A1 cso (6)
			12